# Pre-Calculus Quarter 1 Test 3 Review Answer Key

1) a) $f(1) = (1)^3 - 3(1) + 5$

$= 1 - 3 + 5$

$f(1) = \boxed{3}$

b) $f(-2) = (-2)^3 - 3(-2) + 5$

$= -8 + 6 + 5$

$f(-2) = \boxed{3}$

c) $f(x-2) = (x-2)^3 - 3(x-2) + 5$

$(x-2)(x-2)(x-2) - 3(x-2) + 5$

$(x^2 - 4x + 4)(x-2) \quad -3x + 6 + 5$

$x^3 - 2x^2 - 4x^2 + 8x + 4x - 8 \quad -3x + 6 + 5$

$f(x-2) = \boxed{x^3 - 6x^2 + 9x + 3}$

d) $f(2x) = (2x)^3 - 3(2x) + 5$

$f(2x) = \boxed{8x^3 - 6x + 5}$

e) $f(x+h) = (x+h)^3 - 3(x+h) + 5$

$(x+h)(x+h)(x+h) - 3x - 3h + 5$

$(x^2 + 2xh + h^2)(x+h) - 3x - 3h + 5$

$x^3 + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3 - 3x - 3h + 5$

$f(x+h) = \boxed{x^3 + 3x^2h + 3xh^2 + h^3 - 3x - 3h + 5}$

2) a) $(g \circ f)(x) = g(x-1) = (x-1)^2 = \boxed{x^2 - 2x + 1}$

b) $(g \circ h)(x) = g\left(\sqrt{x^2-16}\right) = \left(\sqrt{x^2-16}\right)^2 = \boxed{x^2 - 16}$

c) $(f \circ g \circ h)(x) = \quad h(x) = \sqrt{x^2-16}$

$\downarrow$

$g\left(\sqrt{x^2-16}\right) = \left(\sqrt{x^2-16}\right)^2 = x^2 - 16$

$\downarrow$

$f(x^2-16) = (x^2-16) - 1 = \boxed{x^2 - 17}$

d) $(f \circ h \circ g)(x) = \quad g(x) = x^2$

$$h(x^2) = \sqrt{(x^2)^2 - 16} = \sqrt{x^4 - 16}$$

$$f\left(\sqrt{x^4-16}\right) = \boxed{\sqrt{x^4-16} - 1}$$

3) a) $$\frac{(x+h)^2 - 3(x+h) - 4 - (x^2 - 3x - 4)}{h} = \frac{x^2 + 2xh + h^2 \boxed{-3x} - 3h - 4 - x^2 \boxed{+3x} + 4}{h}$$

$$= \frac{2xh + h^2 - 3h}{h} = \boxed{2x + h - 3}$$

b) $$\frac{3(x+h) - 2 - (3x-2)}{h} = \frac{3x + 3h \boxed{-2} - 3x \boxed{+2}}{h} = \frac{3h}{h} = \boxed{3}$$

c) LCD: $x(x+h)$

$$\frac{\cancel{x(x+h)}\frac{1}{\cancel{x+h}} - \frac{1}{\cancel{x}}\cancel{x}(x+h)}{\frac{h}{1}\ x(x+h)} = \frac{x - (x+h)}{xh(x+h)} = \frac{-h}{xh(x+h)} = \boxed{\frac{-1}{x(x+h)}}$$

or $\boxed{\frac{-1}{x^2+xh}}$

4) a) $f(x) = 3x - 2$

$y = 3x - 2$

$x = 3y^{-1} - 2$

$x + 2 = 3y^{-1}$

$\frac{x+2}{3} = y^{-1}$

$\boxed{f^{-1}(x) = \frac{x+2}{3}}$

yes one to one

b) $y = x + 20$

$x = y^{-1} + 20$

$x - 20 = y^{-1}$

$\boxed{y^{-1} = x - 20}$

yes 1 to 1

c) $f(x) = \sqrt{3x-6}$

$y = \sqrt{3x-6}$

$x = \sqrt{3y^{-1} - 6}$

$x^2 = 3y^{-1} - 6$

$x^2 + 6 = 3y^{-1}$

$\frac{x^2+6}{3} = y^{-1}$

$\boxed{f^{-1}(x) = \frac{x^2+6}{3}}$

yes 1 to 1

5) a) $\overset{x_1\ y_1}{(2,-4)}$ $\overset{x_2\ y_2}{(-2,7)}$

$$\frac{7--4}{-2-2} = \boxed{\frac{11}{-4}}$$

b) $-3x+4y=12$

$3x-4y=-12$ ← Standard form Slope $= \frac{-A}{B}$

$$m = \frac{-3}{-4} = \boxed{\frac{3}{4}}$$

c) $y+3=2(x-3)$

Point slope form

Slope = 2

$$\boxed{\text{Slope of } \perp = \frac{-1}{2}}$$

d) $y=3(2x-5)$

$y=6x-15$

$$\boxed{\text{Slope} = 6}$$

6) a) $\boxed{y--4=3(x-2)}$ ← Point slope

$y+4=3x-6$

$\ \ -4 \qquad -4$

$\boxed{y=3x-10}$ ← slope intercept

$y-3x=-10$

$-3x+y=-10$

$\boxed{3x-y=10}$ ← Standard form

b) $m = \frac{7--4}{-2-2} = \frac{11}{-4}$

Use either point $(2,-4)$ or $(-2,7)$

$y-7 = -\frac{11}{4}(x--2)$ ← point/slope

$y-7 = -\frac{11}{4}x - \frac{22}{4}$

$\ \ +7 \qquad\qquad +7$

$\boxed{y = -\frac{11}{4}x + \frac{6}{4}}$ ← slope/intercept

$y + \frac{11}{4}x = +\frac{6}{4}$

$4\left(\frac{11}{4}x + y = +\frac{6}{4}\right)$

$\boxed{11x+4y=+6}$ Standard form

7) a) $\cancel{7}x-2$

$f(x)=7x$

$g(x)=x-2$

$\boxed{g(f(x))}$

b) $\frac{12}{\sqrt{x+12}}$

$f(x)=x+12$

$g(x)=\sqrt{x}$

$h(x)=\frac{12}{x}$

$\boxed{h(g(f(x)))}$

c) $(x^4-6)^9$

$f(x)=x^4$

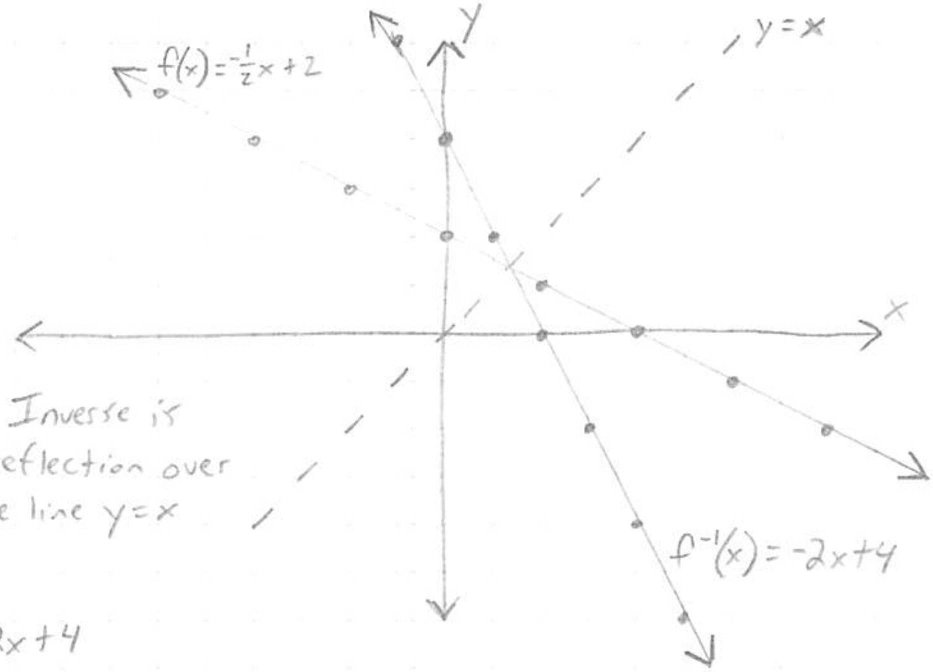
$g(x)=x-6$

$h(x)=x^9$

$\boxed{h(g(f(x)))}$

8) a) $f(x) = -\frac{1}{2}x + 2$

$m = -\frac{1}{2} \quad b = 2$




b) $f(x) = -\frac{1}{2}x + 2$

$y = -\frac{1}{2}x + 2$

$x = -\frac{1}{2}y + 2$

$x - 2 = -\frac{1}{2}y$

$-2(x-2) = y \qquad f^{-1}(x) = -2x+4$

$-2x+4 = y$

9) $f(x) = \sqrt{x-5} \qquad g(x) = x^2+5$

$f(g(x)) = f(x^2+5) = \sqrt{x^2+5-5} = \sqrt{x^2} = x$ ✓

OR

$g(f(x)) = g(\sqrt{x-5}) = (\sqrt{x-5})^2 + 5 = x-5+5 = x$ ✓

$f(g(x)) = g(f(x)) = x$ ✓

10) a) $f(g(2))$

$g(2) = 3$

$f(3) = \boxed{1}$

b) $g(f(0))$

$f(0) = 2$

$g(2) = \boxed{3}$

c) $(g \circ f)(6)$

$f(6) = 0$

$g(0) = \boxed{2}$

d) $(f \circ f)(6)$

$f(6) = 0$

$f(0) = \boxed{2}$

The given value will lead to a y value that becomes the input of the second function of the composition.